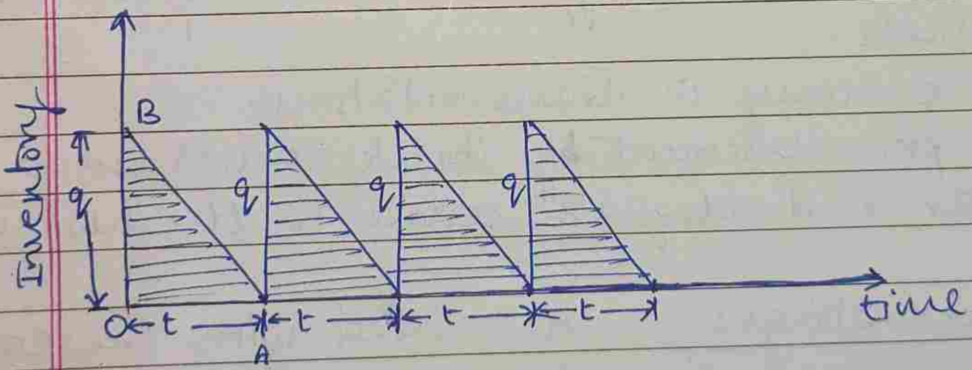
## Some Important models

### Model I

To derive an economic lot size formula and minimum average cost under following conditions:-

(i) Uniform rate of demand is r units per unit time
(ii) production rate is infinite.
(iii) lead time is zero
(iv) $C_1$ = holding cost per unit per unit time
(v) $C_3$ = set up cost per production run
(vi) storages are not allowed.

Here let q be the quantity units produced per production run at interval of time t.




Since demand rate is r units per unit time

∴ Total demand in one run in time $t = rt$

∴ The quantity produced per production run

$$q = rt \quad \text{——— (1)}$$

The cost of holding inventory

$= C_1 (\text{Area of } \Delta OAB) = C_1 \frac{1}{2} qt$

and set up cost $= C_3$

$\therefore$ The total cost per production run of time

$$= \frac{1}{2} C_1 q t + C_3$$

$\therefore$ The average total cost per unit time

$$\boxed{C(q) = \frac{1}{2} C_1 q + \frac{C_3}{t} = \frac{1}{2} C_1 q + \frac{C_3 r}{q}} \quad \text{——— (2)}$$

This equation is known as cost equation.

For min. value of $C(q)$

$$\frac{dC}{dq} = \frac{1}{2} C_1 - \frac{C_3 r}{q^2} = 0$$

or $\frac{C_3 r}{q^2} = \frac{1}{2} C_1$

or $q^2 = \frac{2 C_3 r}{C_1}$

or $q = \sqrt{\left(\frac{2 C_3 r}{C_1}\right)}$

Since $\frac{d^2C}{dq^2} = \frac{2 C_3 r}{q^3}$ is +ve for $q = \sqrt{\left(\frac{2 C_3 r}{C_1}\right)}$

$\therefore$ $C(q)$ given by (2) is minimum for

$$\boxed{q = q^* = \sqrt{\left(\frac{2 C_3 r}{C_1}\right)}} \quad \text{——— (3)}$$

This is the economic lot size formula.

Now from (1) the optimum value of t is given by

$$t = t^* = \sqrt{\left(\frac{2C_3}{C_1 r}\right)}$$

and from (2) the minimum cost per unit time is given by

$$C_{min} = \frac{1}{2} C_1 \sqrt{\left(\frac{2C_3 r}{C_1}\right)} + C_3 r \sqrt{\left(\frac{C_1}{2C_3 r}\right)}$$

or $$\boxed{C_{min} = \sqrt{(2 C_1 C_3 r)}} \quad \text{———} \quad (4)$$

If $C_1$ and $C_3$ are constants then the minimum cost per unit time is proportional to the square root of the demand rate.