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(Topology)

Definitions and Properties
of
Topological Spaces

(1)

Topological Space :— Let X be a set and $\mathcal{J}$ be the collection of subsets of X satisfying the following condition:

(1) $\phi \in \mathcal{J}$, $X \in \mathcal{J}$

(2) If $G_1 \in \mathcal{J}$, $G_2 \in \mathcal{J}$ then $G_1 \cap G_2 \in \mathcal{J}$

(3) If $G_\lambda \in \mathcal{J}$ $\forall \lambda \in \Delta$ where Δ is arbitrary set then $\cup \{G_\lambda : \lambda \in \Delta\} \in \mathcal{J}$

Then $\mathcal{J}$ is called a topology for X and $(X, \mathcal{J})$ is the topological space. The members of $\mathcal{J}$ are called $\mathcal{J}$- open or open sets.

The empty set ϕ and the whole space X are open.

The intersection of open sets is open.

The arbitrary union of open sets is open.

Ex/175, Ex2/177

<u>Indiscrete Topology</u> / 177 A topology is said to be indiscrete if it consists of the empty set ϕ and whole space X, necessary a topology for X. It is also known as trivial topology.

<u>Discrete Topology</u> / 177 Let D be the collection of all subsets of X, then D is a topology for X called the discrete topology.

<u>Comparison of Topologies</u>:— Let $(X, \mathcal{J}_1)$ & $(X, \mathcal{J}_2)$ be two topological spaces. Then $\mathcal{J}_1$ is coarser, weaker or smaller than $\mathcal{J}_2$ or $\mathcal{J}_2$ is finer, stronger or larger than $\mathcal{J}_1$ iff $\mathcal{J}_1 \subset \mathcal{J}_2$.

Now, if either $\mathcal{J}_1 \subset \mathcal{J}_2$ or $\mathcal{J}_2 \subset \mathcal{J}_1$ then $\mathcal{J}_1$ & $\mathcal{J}_2$ are comparable.

Closed Sets :— Let (X, J) be a topological space. Let F be a subset of X. Then F is said to be closed if and only if its complement F' is open. Thus,

ϕ is open $\Rightarrow \phi' = X$ is closed.

X is open $\Rightarrow X' = \phi$ is closed.

Neighbourhood :— Let (X, J) be a topological space and let $x \in X$. A subset N of X is said to be a nbd. of x iff $\exists$ an open set G such that $x \in G \subset N$.

Base for a Topology :— Let (X, J) be a topological space. A non-empty collection $\mathcal{B}$ of subsets of X is a base for J iff

(i) $\mathcal{B} \subset J$ (ii) $\forall x \in X$ and each nbd. N of x $\exists$ some $B \in \mathcal{B}$ such that $x \in B \subset N$.

Second Countable Space :— Let (X, J) be a topological space. The space is said to be second countable iff $\exists$ a countable base for J.

Limit Point :— Let (X, J) be a topological space and let $A \subset X$. A point $x \in X$ is called a limit point or an accumulation point of A iff every nbd. of x contains a point of A other than x.

The set of all limit points of A is called the derived set of A and denoted by $D_X(A)$ or simply $D(A)$.

Adherent Point :— Let (X, J) be a topological space and let $A \subset X$. A point $x \in X$ is called Adherent point of A iff every nbd. of x contains a point of A.

. The set of all adherent points of A is denoted by Adh(A).

Housdorff Space :— A topological space $(X, \mathcal{T})$ is said to be Hausdorff space or a separated space or a T_2 - space iff for all pair of distinct points x, y of X $\exists$ disjoint nbds N of x and M of y such that $N \cap M = \phi$.

Closure :— Let $(X, \mathcal{J})$ be a topological space and $A \subset X$. The intersection of all closed supersets of A is called the closure of A and is denoted by $\bar{A}$ or $C(A)$ or $Cl(A)$.

Interior Point :— Let $(X, \mathcal{J})$ be a topological space and let $A \subset X$. Then $x \in A$ is said to be an interior point of A iff A is a nbd. of x i.e. iff $\exists$ an open set G such that $x \in G \subset A$.

The set of all interior points of A is called the interior of A and is denoted by A° or A^{i} or $i(A)$ or $int(A)$. Also A contains all its interior points i.e. $A^{\circ} \subset A$

Illustration $\frac{}{176}$, $\frac{Ex\ 11}{181}$, $\frac{Thm\ 1}{183}$, $\frac{Thm\ 2}{184}$,

Co-countable Topology — A topological space $(X, \mathcal{J})$ is said to be a co-countable topology if $\mathcal{J}$ consists of all those subsets of X whose complements are countable sets together with the empty set.

Ref — Krishna Pub. Meerut.