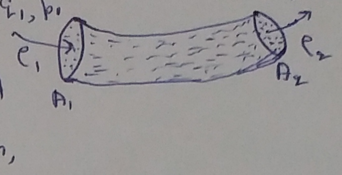
Fluid Dynamics — H-4050 (Maths)

Bernoulli's equation (Stream tube method):

Consider a stream tube with A_1 and A_2 as its two cross-sections. Let ρ_1, p_1, q_1 and ρ_2, p_2, q_2 be the density, pressure and the velocities at its two ends. Since the velocity normal to the curved surface of the stream tube is always zero therefore the flow will take place only through A_1 and A_2. From the law of conservation, we have



$$\rho_1 A_1 = \rho_2 A_2 = k \text{ (Constant)} \quad \text{— (1)}$$

The net force on the tube due to pressure is

$$p_1 A_1 - p_2 A_2 = \frac{p_1}{\rho_1}(\rho_1 A_1) - \frac{p_2}{\rho_2}(\rho_2 A_2)$$

$$\Rightarrow \quad p_1 A_1 - p_2 A_2 = k\left(\frac{p_1}{\rho_1} - \frac{p_2}{\rho_2}\right)$$

The difference of the forces acting on the two cross-sections must be equal to the gain in the energy of the fluid passing through the tube. Thus, we have

$$\frac{p_1}{\rho_1} - \frac{p_2}{\rho_2} = k_2 - k_1$$

or,

$$\frac{p_1}{\rho_1} + k_1 = \frac{p_2}{\rho_2} + k_2 = \text{Constant} \quad \text{— (2)}$$

where k_1 and k_2 are the sum of kinetic $\left(\frac{1}{2}q^2\right)$ and potential (Ω) energies per unit mass of the fluid entering and leaving the tube. Thus, we have

$$\frac{p}{\rho} + \frac{1}{2}q^2 + \Omega = \text{Constant} \quad \text{— (3)}$$

Again, let the cross-sections A_1 and A_2 of the stream tube tends to zero such that it reduces to a stream line, even then equation (3) holds. Hence the equation holds along a stream line.

Reference: Fluid dynamics by Krishna's Educational Publishers, Meerut.